Dispersive optical bistability in one-dimensional doped photonic band gap structures

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(Received 30 July 1996; revised manuscript received 31 October 1996)

We introduce a Kerr-type nonlinear defect layer in the center of one-dimensional photonic band gap structure. For linear parts, we use transfer matrix technique, and for the nonlinear layer, we solve the Maxwell equation numerically. When incident light intensity varies, a typical S-shape curve of the transmitted light intensity is obtained. In this case, an optical bistability is produced by dynamic shifting of the defect mode frequency, not dynamic shifting of the band edge, which is different from the case of periodic nonlinear superlattices. It is also found that when the linear defect mode frequency moves from the center of the gap to the edge of the gap, the threshold intensity needed for the bistability increases rapidly. [S1063-651X(97)05204-5]

PACS number(s): 42.25.Bs, 78.66.−w, 42.65.Pc

Recently, there has been increasing interest in photonic band gap (PBG) structures [1], which have forbidden bands prohibiting a certain range of frequencies of light waves from propagating through them. Many theoretical and experimental studies focus on the linear case. When nonlinearity is introduced, it can cause many amusing phenomena, for example, optical limiting and switching [2], optical diodes [3], and optical bistability [4]. The above nonlinear phenomena are caused by the dynamic shifting of the band edge, i.e., the incident light frequency is tuned to near the band edge, and when the incident light intensity changes, it causes a change of effective refractive index, and hence a change of the band edge too. However, this kind of single distributed feedback (DFB) structure needs a large number of periods to obtain a sharp and strong resonance. He and Cada [5] proposed a combined structure which is composed of a DFB structure and a phase-matching layer placed in a Fabry-Pérot (FP) cavity. They developed a nonlinear matrix transfer method to study optical bistability in the combined DFB-FP structure at frequencies near the band edge, and found that it can have a much lower threshold value for the bistability than a single DFB structure with a comparable total thickness.

When a defect is introduced in the PBG structure, it can create a donor or an acceptor mode in the forbidden band [6], which is similar to the case of a semiconductor. The defect mode frequency depends on the refractive index and volume of the defect. In the one-dimensional (1D) case [7], the donor mode frequency decreases when the refractive index of the defect layer increases. It has been found that when the defect layer has the lowest refractive index, the electric field in it is the largest, and nearly 20 times larger than the incident field. Thus if the defect has nonlinearity such a structure can enhance the nonlinear effect greatly.

For a PBG structure doped with linear dielectric materials, if a light wave is tuned at the defect mode frequency, it can pass through it with almost no reflection. If the light frequency ω is tuned in the gap (not at the defect mode frequency), the field will decrease exponentially in the structure. With a Kerr-type nonlinearity existing in the defect layer, the defect mode frequency will now change with the local light intensity, and hence the incident light intensity. This means if we tune ω near to the defect mode frequency the doped structure can produce a positive or negative (depending on the sign of nonlinearity and whether ω is bigger or smaller than the defect mode frequency) feedback on the incident light. We have known that a bistability can appear in an optical system with a positive feedback [8], and optical bistability systems have many applications. For example, they can be used as optical logic elements, memory elements, or optical transistors [8,9]. Therefore it is very interesting to investigate the bistability in the doped 1D PBG structure.

We use a quarter-wavelength stack as our 1D PBG structure, which is a multilayer stack of dielectric materials in which alternating layers have a lower refractive index n_1 (denoted as A), and a higher refractive index n_2 (denoted as B). Thicknesses for the two kinds of layers are such that $d_1=\lambda_0/4n_1$ and $d_2=\lambda_0/4n_2$, with λ_0 as the free-space wavelength. This kind of structure can create a band gap with center frequency $2\pi c/\lambda_0$. We now replace the center layer A by another nonlinear layer C (here, C may have different thickness from A). The doped structure has the form $\ldots ABABCBABA \ldots$, which can be considered to be composed of three parts linked together by two nearest-neighbor B layers of the nonlinear C layer. These three parts are the left and the right linear parts $\ldots ABABA \ldots$, and the nonlinear layer C. The left linear part is supposed to lie in the coordinate space, $z_1<z<z_{N+1}$. Electric field in the structure can be expressed by $E(z,t)=E(z)e^{-i\omega t}$ with

$$E(z)=\begin{cases} s\exp(ik_0z)+\gamma\exp(-ik_0z) & \text{if } z<z_1 \\ f_j\exp[ik_j(z-z_j)]+b_j\exp[-ik_j(z-z_{j+1})] & \text{if } z_j<z<z_{j+1} \\ t\exp[ik_s(z-z_{N+1})]+r\exp[-ik_s(z-z_{N+1})] & \text{if } z>z_{N+1}, \end{cases}$$

where $j=1,2,3,\ldots,N$ is the layer index, the wave vector $k_j=2\pi n_j/\lambda_0$ with λ_0 as the wavelength of incident light in free space, and z_j is the position of the jth interface. Both f_j and b_j are the coefficients in the jth layer to be determined by the boundary conditions that the tangential components of $E(z)$ and its derivative over z should be continuous at the boundary. Therefore we have the recurrence relations

$$\begin{bmatrix} f_{j-1} \\ b_{j-1} \end{bmatrix}=\begin{bmatrix} a_{11}^j & a_{12}^j \\ a_{21}^j & a_{22}^j \end{bmatrix}\begin{bmatrix} f_j \\ b_j \end{bmatrix}, \tag{1}$$

in Eq. (1), for $1\leqslant j\leqslant N$,

$$a_{11}^j=\frac{1}{2}\exp(-ik_{j-1}d_{j-1})\left(1+\frac{n_j}{n_{j-1}}\right),$$

$$a_{12}^j=\frac{1}{2}\exp(-ik_{j-1}d_{j-1})\left(1-\frac{n_j}{n_{j-1}}\right),$$

$$a_{21}^j=\frac{1}{2}\exp(ik_{j-1}d_{j-1})\left(1-\frac{n_j}{n_{j-1}}\right),$$

$$a_{22}^j=\frac{1}{2}\exp(ik_{j-1}d_{j-1})\left(1+\frac{n_j}{n_{j-1}}\right), \tag{2}$$

with $d_j=z_{j+1}-z_j$ as the thickness of the jth layer. For $j=1$, we have $f_0=s$, $b_0=\gamma$, $d_0=z_1$, and n_0 as the refractive index in the semi-infinite space $z\leqslant z_1$. Finally when $j=N+1$, we have $f_{N+1}=t$, $b_{N+1}=r$ for the left linear part, and n_{N+1} is the refractive index in the space $z>z_{N+1}$.

Considering correspondence between the 1D Maxwell and Schrödinger equations, we can define the total transfer matrix for N layer films [10],

$$M_N=\left\{\prod_{j=1}^{N+1}\begin{bmatrix} a_{11}^j & a_{12}^j \\ a_{21}^j & a_{22}^j \end{bmatrix}\right\}, \tag{3}$$

which links electric fields outside the structure,

$$\begin{bmatrix} s \\ \gamma \end{bmatrix}=\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}\begin{bmatrix} t \\ r \end{bmatrix}. \tag{4}$$

The same procedure can be done for the right linear part except that there exists no reflected wave in the right outside of it, i.e., the $r=0$ in Eq. (4) right now. The details can be found in Ref. [11].

Now, we consider the nonlinear layer C. For a Kerr-type nonlinearity, the effective index of refraction is

$$n^2=n_0^2+\chi_3|E(z)|^2, \tag{5}$$

with χ_3 as a small nonlinear coefficient. Assuming the electromagnetic field has the form $E(z)e^{-i\omega t}$, the Maxwell equation becomes

$$\frac{d^2E(z)}{dz^2}=-\frac{\omega}{c^2}[n_0^2+\chi_3|E(z)|^2]E(z). \tag{6}$$

Here c is the light velocity in free space. Introducing the dimensionless coordinate $\xi=z/\lambda_0$, with λ_0 as the light wavelength in free space, we can rewrite Eq. (6) as follows:

$$\frac{d^2E(\xi)}{d\xi^2}=-4\pi^2[n_0^2+\chi_3|E(\xi)|^2]E(\xi). \tag{7}$$

Discretizing Eq. (7), we get

$$\begin{aligned} E(\xi-\delta\xi)=&\,2E(\xi)-E(\xi+\delta\xi) \\ &+\delta\xi^2\{-4\pi^2[n_0^2+\chi_3|E(\xi)|^2]E(\xi)\} \\ &+O(\delta\xi^3). \end{aligned} \tag{8}$$

Using the boundary conditions, we can link the left and right linear parts and the nonlinear layer together. For a given transmitted field, using Eq. (4), we can first obtain the field transmitted through and reflected on the right boundary of the nonlinear layer. Then using Eqs. (4) and (8), we can finally obtain the field incident on the doped structure.

We take $n_1=1.5$, $n_2=2.0$, $n_3=1.0983$, d_3 (the thickness of defect layer) $=3.1d_1$, $\omega=0.236$ (in units of $2\pi c/n_1d_1$), $\chi_3=0.01$, and the number of total layers is 41 (for the linear part, $N=19$). The numerical calculation result for the relation between the incident (I_{in}) and transmitted intensity (I_{out}) is shown in Fig. 1, from which an S-shape response of the doped 1D structure can be seen clearly. When I_{in} increases slowly from zero, I_{out} first increases slowly, and when I_{in} reaches a threshold value I_1 (about 4.7), I_{out} will jump to a higher value. Then, it increases slowly again with increasing of I_{in}. When I_{in} decreases slowly from a value higher than I_1, I_{out} decreases slowly from the high value. When I_{in} reaches I_1, I_{out} will not jump back to the lower value, but continues to decrease slowly until I_{in} reaches another threshold I_2 (about 0.17), at which I_{out} jumps back to a lower value. Then, it decreases slowly with decreasing of I_{in}. Thus it is obvious that the doped 1D stack can produce an optical bistability.

In the beginning, even ω is tuned to near the linear defect mode frequency Ω_0 [determined only by the n_0 in Eq. (5)]; the transmission coefficient is still smaller since I_{in} is lower. With increasing of I_{in}, the nonlinear effect that happened in the doped layer changes the defect mode frequency Ω, and makes it move to ω (positive feedback), which increases the transmission coefficient slowly. Once I_{in} reaches I_1, the nonlinear effect makes Ω almost equal to ω, and so the transmission coefficient has a steplike increase. After that, further

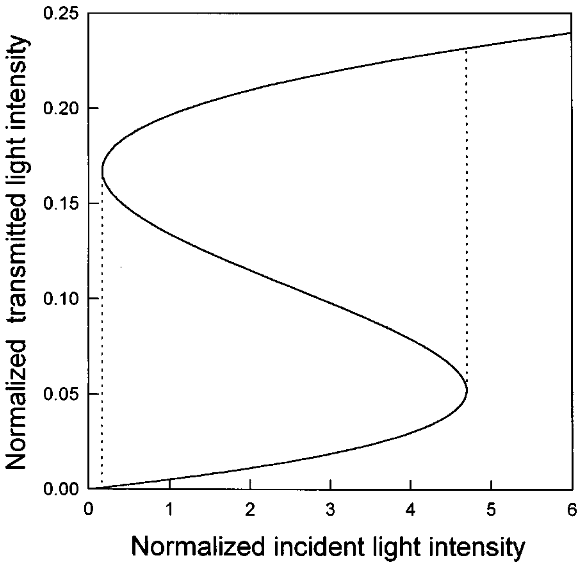

FIG. 1. The normalized transmitted light intensity varies with the normalized incident light intensity.

increasing of I_{in} will cause Ω to slowly move away from ω. This negative feedback will hold Ω in the vicinity of ω. So the I_{out} will increase slowly with increasing of the I_{in}. In this region of I_{in}, the doped structure works as an optical limiter. Oppositely, when I_{in} decreases from a value higher than I_1, since the structure has a higher transmission coefficient now, the field in the nonlinear layer is also high, and the nonlinear effect keeps Ω in the vicinity of ω until I_{in} decreases to another threshold value I_2, which is lower than I_1. At this time, the nonlinear effect cannot hold Ω in the vicinity of ω any more, so the transmission coefficient drops to a lower value.

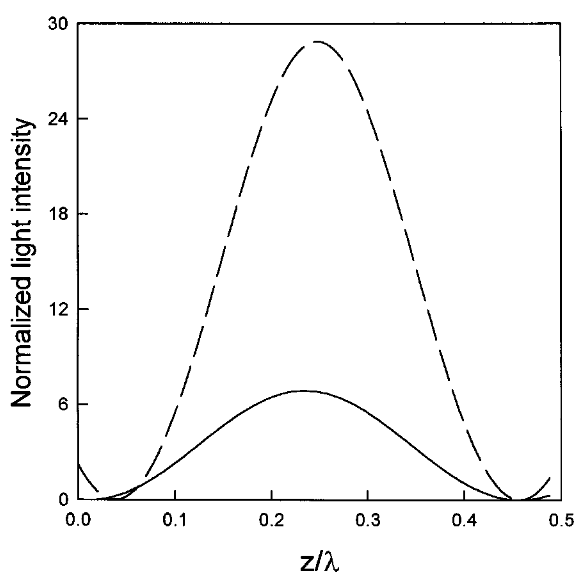

FIG. 2. The normalized light intensity in the nonlinear layer for the case of increasing the normalized incident light intensity I_{in}. Solid and dashed lines correspond to $I_{\text{in}}<I_1$ and $I_{\text{in}}>I_1$, respectively. Difference of incident light intensities in both cases is $\triangle I_{\text{in}}\approx 0.004\,72$.

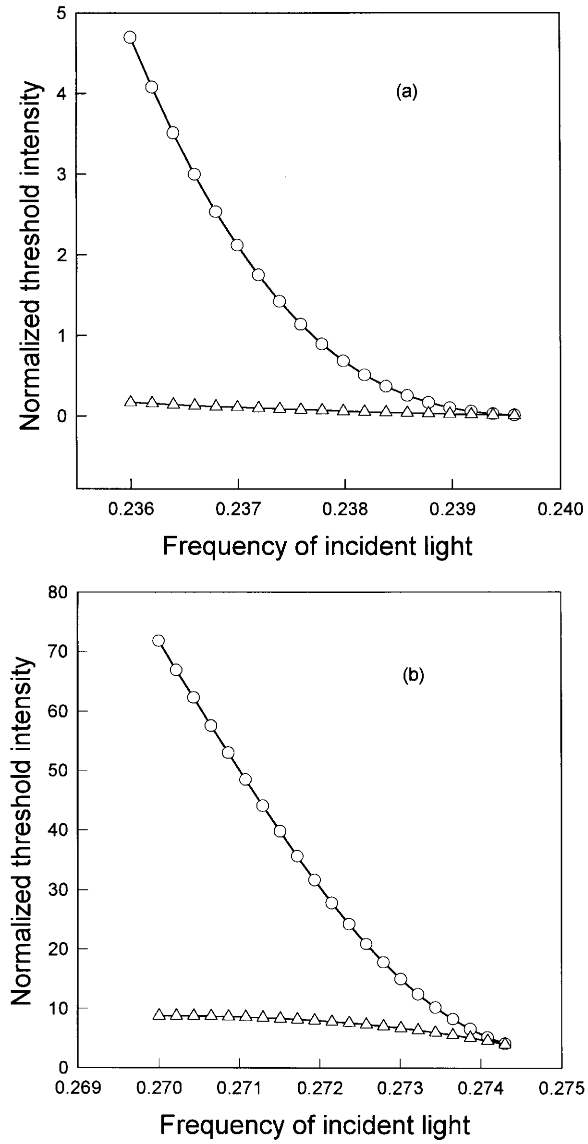

FIG. 3. The normalized threshold values vary with the incident light frequency (in units of $2\pi c/n_1 d_1$). The circles and triangles represent the threshold values I_1 and I_2, respectively. (a) For the doped structure; (b) for the single DFB structure.

To explain our discussion above more clearly, we plot the light intensity in the nonlinear layer C in Fig. 2. The solid (dashed) line represents the light intensity in the nonlinear layer when I_{in} is close to, but smaller (larger) than I_1. The difference (ΔI_{in}) of the two incident light intensities corresponding to the case of solid and dashed lines is very small, $\Delta I_{\text{in}}\approx 0.00472$. From Fig. 2, we can see that the light intensity in the nonlinear layer has a bigger increase when I_{in} increases a ΔI_{in}, and so the effective refractive indices in Eq. (5) are much different from each other. The Ω is still far from ω for the solid line case, but it is almost equal to ω for the dashed line case, which means the doped structure is in different states, high or small transmission state.

Now, we investigate variation of the threshold value I_1 and I_2 with ω. In Fig. 3(a), we see I_1 increases rapidly when ω is tuned away from Ω_0, while I_2 remains almost unchanged, which can be naturally explained as follows. Let us

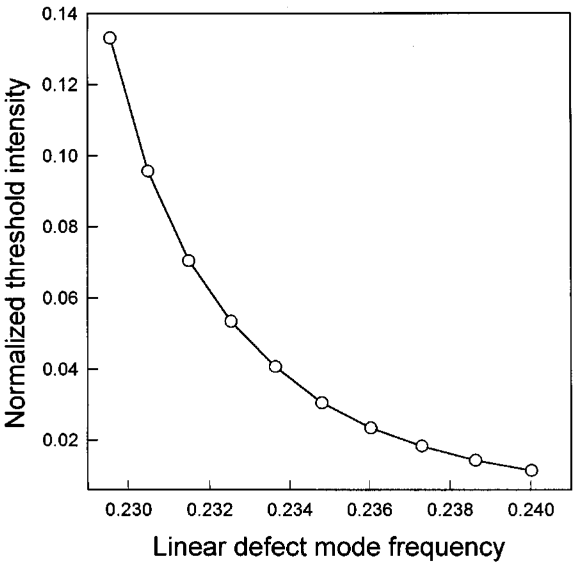

FIG. 4. The normalized threshold intensity needed for switching varies with the linear defect mode frequency Ω_0 (in units of $2\pi c/n_1 d_1$).

first look at the variation of I_1. When $I_{\rm in}$ increases from zero to I_1, because the transmission coefficient is smaller, the electric field intensity $|E(\xi)|^2$ in the nonlinear layer is small too, and the nonlinear effect due to Eq. (5) cannot become stronger. Thus if ω deviates much from Ω_0 a larger $I_{\rm in}$, according to Eq. (5), should be needed to produce a nonlinear effect that is stronger enough to make Ω move to near ω. That, of course, means a larger I_1 is needed. However, the situation is completely different for I_2 because in this case, the nonlinear effect is larger, and only a smaller $I_{\rm in}$ is needed to keep Ω almost equal to ω. This phenomenon is independent of how far away ω deviates from Ω_0. So, I_2 stays smaller and is almost invariable with changing of ω.

For comparison, we have also investigated the optical bistability in a single DFB structure with a total of 40 layers having alternate linear (n_1) and nonlinear (n_2) refractive indices. In our calculation, we have taken $n_1=1.5$, $n_2^2=(2.0)^2+0.01|E|^2$, and the numerical result is shown in Fig. 3(b). It is found that the threshold values of the bistability at different frequencies now become much higher than that of the doped structure.

It is interesting to study dependence of the bistability on the variation of Ω_0. It is found that the threshold value increases with Ω_0 moving from the center of the gap to the edge of the gap (Fig. 4), which is obviously related to the lower field enhancement in the case of Ω_0 moving.

We have studied the nonlinear effect in a doped 1D PBG structure. We found that when a Kerr-type nonlinearity exists, the doped structure will exhibit a bistability. In this case, the optical bistability is caused by the dynamic shifting of Ω, not the conventional dynamic shifting of the band edge. From a comparison between the band edge resonance and our defect mode, it is known that there is no essential difference between these two methods of realizing the optical bistability. The threshold value of the bistability for the doped structure is much lower than that for the single DFB structure, but compatible with that for the combined DFB-FP structure proposed by He and Cada [5]. In our calculation, we use the Kerr-type nonlinearity with positive sign. For a Kerr-type nonlinearity with negative sign, we just need to tune the incident light frequency at the other side of Ω_0.

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